

B.Sc. Semester - 5 (Remedial) (NEP-2020) Examination

March/April-2026

MATH: Abstract Algebra -1 & Complex Analysis -1 (Major-11)

Time: 2:00 Hours

Marks:50

Instructions:

1. All questions are compulsory.
2. Figures to the right indicate marks.

- Que.1 Answer any TWO. (10)
- (1) Define binary operation and associative binary operation. Is the operation $*$ on $G = \mathbb{R}$, defined by $a * b = 1 + a + b + ab \quad \forall a, b \in G$, an associative binary operation on G ? Justify.
 - (2) Prove that the set of cube roots of unity form a commutative group with respect to the binary operation of multiplication of complex numbers.
 - (3) Prove that an infinite cyclic group has exactly two generators.
- Que.2 Answer any TWO. (10)
- (1) Define a symmetric group and alternating group. Prepare a group table for alternating group A_3 of symmetric group S_3 in usual notations.
 - (2) Find $f^{-1}g^{-1}$ and $g^{-1}f^{-1}$ for the following permutations f and g of S_5 .

$$f = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 3 & 2 & 4 & 5 & 1 \end{pmatrix} \text{ and } g = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 5 & 2 & 4 & 3 \end{pmatrix}.$$
 - (3) State and prove Lagrange's theorem for a subgroup of a finite group.
- Que.3 Answer any TWO. (10)
- (1) Is $f(z) = \begin{cases} \frac{(\bar{z})^2}{z} & \text{when } z \neq 0 \\ 0 & \text{when } z = 0 \end{cases}$ differentiable at origin? Justify.
 - (2) Prove that $u(x, y) := \frac{y}{x^2 + y^2}$ is a harmonic function. Also find its harmonic conjugate.
 - (3) Suppose that $f(z) = u(x, y) + i v(x, y)$, $z_0 = x_0 + i y_0$ and $w_0 = u_0 + i v_0$. Then $\lim_{z \rightarrow z_0} f(z) = w_0$ if and only if $\lim_{(x, y) \rightarrow (x_0, y_0)} u(x, y) = u_0$ and $\lim_{(x, y) \rightarrow (x_0, y_0)} v(x, y) = v_0$.
- Que.4 Answer any TWO. (10)
- (1) Let $f(z) = \pi e^{(\pi \bar{z})}$ and C is the boundary of the square with vertices at the points $0, 1, 1 + i$ and i . If the orientation of C is taken in the anticlockwise direction then find $\int_C f(z) dz$.
 - (2) If $f(z_0) := \int_C \frac{z^2 + 2z + 5}{z - z_0} dz$, $C : |z| = 3$, then find $f(1)$, $f(-2)$, $f'(i)$, $f'(-2)$ & $f(t)$ for $|t| > 3$.
 - (3) State and prove Cauchy integral formula for a function f which is analytic everywhere within and on a simple closed contour C .
- Que.5 Answer any TWO. (10)
- (1) Let G be a group and H be its subgroup. If index of H in G is 2 then prove that H is a normal subgroup of G .
 - (2) Prove: The composition of any two disjoint cycles of S_n is commutative.
 - (3) State and prove necessary and sufficient condition for a function to be analytic at a point.
